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Dedicated to mathematics in general and to the following aims in particular: (1) a study of the common problems of secondary and collegiate mathematics teaching, (2) a true valuation of the disciplines of mathematics, (3) the publication of high class expository papers on mathematics, (4) the development of greater public interest in mathematics by the publication of authoritative papers treating its cultural, humanistic and historical phases.

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VALUABLE CORRELATIONS

The mind that, through years of scientific training in the materials of mathematics, has become habituated to the practice of arriving at its conclusions from accepted data solely by chains of necessary implication will be prone to attack every problem, whether mathematical in character or not, by use of the necessary-implication chain.

Profoundly influential in preparing such mind for the rigorous application of scientific method is that phase of mathematical logic which deals with the truth of "necessary and sufficient conditions". We are persuaded that often do we lean too heavily upon the assumption that logical phases of the mathematics should be deferred until the student is more mentally mature, so that, not infrequently we miss the very period of growth when his sinewy young powers are best adapted to such athletic exercises. He is taught to reason: If x is true, and if y is true, then must z be true. Is he also taught to inquire: Are x and y true if z is true? Yet the steady coordination of a given process with its inverse is fundamental and vital in a multitude of investigations that deal with the implications of nature's laws. There is a real disciplinary connection between the process of mind that habitually asks a question of the type, "If a circle implies that chords of equal length are equi-distant from the centre, does it also imply that equal distances from the center imply equal chords", and the process of mind that inquires, "If magnetism is produced by electricity, is electricity producible from magnetism?"

Just as in mathematics, so in the well-nigh unlimited fields of nature's operations, before truth of relation may be completely garnered it is often necessary to inquire: Are the given conditions necessary to certain effects? Are they sufficient for them, but not necessary? Are they neither sufficient nor necessary? Are they both necessary and sufficient?—S. T. S.

EDITORIAL PERSONNEL OF THE MATHEMATICS NEWS LETTER

Acting upon a conviction that the steadily growing group of News Letter readers, already nation-wide in scope, are entitled to know something about the personnel of our editorial Board, we have gathered some interesting biographic facts of which the following are offered as a first installment.—S. T. S.

JAMES McGIFFERT

After receiving Civil Engineering degree at Rensselaer Polytechnic Institute, he attended John Hopkins University, but was called back to Rensselaer to teach. Obtained leave of absence for one year to go to Harvard University, where he obtained degrees of Bachelor and Master of Arts. He received his Ph.D. from Columbia University during teaching time at Rensselaer. His work at Rensselaer is entirely in the graduate department, except for consultation hours held daily for undergraduates.

It is interesting to note that Dr. McGiffert became mathematically inclined in early childhood, when his grandfather taught him cube root as a wee chap, his father having died when he was two years old. The grandfather was a private tutor to William Astor, Hamilton Fish, and other noted men in New York a long time ago, and his love for mathematics, Latin and Greek was inherited by Dr. McGiffert, who says that teaching mathematics is a lot of fun.

CLARENCE DeWITT SMITH

Was born August 9, 1891, at Jackson, Miss., reared in Rankin County, Miss.; B. A. degree Miss. College, 1915; taught in Miss. High Schools three years; Teaching Fellow in mathematics Miss. College, 1914-15; Head Mathematics Dept. Bolton High School, Alexandria, Louisiana, 1915-17; Head of Mathematics Dept., Louisiana College, 1917-1930; Graduate Study, Mississippi College, 1915; attended the University of Chicago, 1916; World War, 1917-19, discharged with the rank of First Lieutenant F. A., U. S. A.; member Committee on Curriculum which prepared Louisiana College for admission to the Southern Association of Colleges, 1922; Graduate Study, University of Iowa, 1924-27; Master's Degree in mathematics, 1925; Doctor's Degree in mathematics, 1928; Head of Mathematics Department, State College, Mississippi, 1930.

Active participation in American Mathematical Society and the Mathematical Association of America; Co-organizer of Kappa Mu Epsilon, National Honorary Fraternity in mathematics; member of Sigma Xi, honorary fraternity devoted to research; Past Master, Free and Accepted Masons; Editorial Staff, Mathematics News Letter; Listed in American Men of Science.

Publications

Tchebycheff Inequalities—Iowa University Library.

On Generalized Tchebycheff Inequalities in Mathematical Statistics—*American Journal of Mathematics.*

Problems—*American Mathematical Monthly.*

Placement and Performance in Freshman Mathematics—*Mathematics Teacher.*

Short Papers—Mathematics News Letter.

Review Exercises for Freshmen.

Lessons in Freshman Mathematics.

College Algebra.—***Continued on page 22.***

THE ORIGIN OF A MATHEMATICAL CONCEPT

By GLENN JAMES
University of California at Los Angeles

A couple of years after I began to teach, a big husky Michigan boy, in his thirties, entered my Plane Geometry class and also my beginning Algebra. He had failed this algebra three times and had been pronounced utterly impossible by his former teachers. In fact he could not get a tutor because his teachers felt it would just be stealing his money. But he had not taken the Geometry before so I decided to put up a fight for him in that subject. The first serious proposition I tried to get him to prove was that two triangles are congruent if two sides and the included angle of one of them are equal to two sides and the included angle of the other. I was determined that he should understand this proof and he was even more determined that he would. But all my efforts to explain the proof geometrically were fruitless. He just sat there with a dazed hungry look and shook his head. Student after student went to the blackboard and proved the proposition in their own peculiar ways. All the conventional tricks of the trade that I knew were exhausted. Still he shook his honest head.

Necessity forced me to a new line of attack, which happened to be an inquiry into his past history. Having learned that he grew up on a farm, I drew rail fences on the blackboard for the sides of the triangles and said, "Now, Mr. Sprague, you and I will fasten these corners together; brace this pen well, then jack it up and move it over on top of the other one . . ." Thus I went through the proof using the rail-pen language throughout. I had not quite finished when he nodded that he understood. Immediately I sent him to the blackboard to repeat the proof. To my surprise, he erased my rail-pens, drew triangles and used the language of Geometry, giving the proof correctly. The symbols of Mathematics had taken on life-meaning for him. A great breach had been made in the barrier that had separated his life of thoughtful experience from the coveted world of Mathematics. His determination did the rest. After this incident he received no more special attention, yet did fair work in geometry, slowly cleared up his algebra, and within seven months was leading a class in Solid Geometry, under another instructor. I review this case because it is tragically typical. Mr. Sprague, like most men, had thought a great deal in terms of rail-fences and other concrete things of every-

day life but did not know that Mathematics is just certain of these thoughts and similar thoughts of other men who have lived during the long past. To him Mathematics had been a collection of undefined symbols "which have no meaning except that which their use in Mathematics gives them." To him Mathematics had been something mystical that some men understood and had found to fit our world in the way of solving practical problems. He had felt that to understand it required a sort of hyper-sense which he did not possess. Indeed it would take more than a hyper-sense to understand a science whose symbols take their meaning only from their use whereas their use cannot have meaning until they do.

Unfortunately this mistaken view of Mathematics which I will characterize as 'symbolitus,' is all too common. Second only to laziness, it is the major cause of failures in our subject. Usually when a student says "Mathematics is just naturally hard for me," he has an infection of symbolitus. The most harmful effect of this disease is its placing a premium upon too credulous minds. Far too many students who have made A's and B's in Mathematics do not know the real meaning of its theorems. For many years I have been asking the boys in my Trigonometry classes for the mechanical significance of the theorem that two triangles are congruent when their three sides are equal. Only about one out of ten realizes that this theorem means that given three sides not more than one triangle can be constructed; in other words, that if you fix the lengths of its three sides a triangle is stable. Yet most of them, when small boys, made crude benches or tables with their mother's butcher knives and their father's hammers, and found out that they had to use diagonal braces to make their contraptions stable. Some of this sort of students later became composers of a formal logic of the symbolitus type, that wastes space in our magazines and libraries and creates prejudice against mathematics proper. Is it any wonder that mathematics is being replaced in our schools by subjects that are supposed to be more humanistic?

This rail-pen incident is the most significant link in a long chain of observations that binds me to the conviction that the concepts of Mathematics have grown and are growing right out of experience. In this respect Mathematics does not differ from other sciences. However, we have not had the need of laboratories that they have. Their laboratories serve our purposes very well, especially do those of Physics and Chemistry. Moreover, workaday life itself is a veritable Mathematical laboratory. Indeed considerations of relative size and

form and the use of the sort of logic we employ in Mathematics are essential parts of most of our daily experiences. That is, everyone is more or less mathematical. In fact any normal industrious student could pass the elementary courses in mathematics if they were presented from the experiential standpoint. The little boy who puts his marbles in a bag, crams the bag in his pocket and goes out to play convinced that the marbles are in his pocket is reasoning just exactly as the profound mathematician is when he says, "If A is in B and B is in C, then surely A is in C." The fact that the letters A, B, C are general, that is, may stand for marbles, bag and pocket, or perhaps cookies, jar and kitchen, or what not, does not alter the essential nature of the argument. The use of these letters is merely a generalization such as the boy has frequently made before he even knew how to play marbles.

Many of the earliest sensory responses of a newly born child are the beginnings of very complex and exceedingly important mathematical concepts. Consider, for example, the concept of a maximum. By a maximum we mean the greatest of a certain set, the biggest duck in the puddle, if you choose to say it that way. Before a child is a week old he has perceived which of the various liquids that he has tasted is most desirable to him. Throughout his babyhood he is occupied hourly with choosing that which tastes best, is most brilliant, or in some way most pleasing to him. As he grows older he becomes interested in a more varied set of comparisons, the tallest man, the highest point of a mountain, the most efficient methods of play or work, the most profitable way of running a business and so on *ad infinitum*. All of these specific instances have in common the characteristic that some one of each group exceeds all the others in that group. This abstract characteristic we call a maximum. Every time one passes over the summit of a mountain he passes over the maximum point of that road. We may as well forget the width of the road and think of it as a curved line since the width has nothing whatever to do with the height above sea level. In Calculus we ordinarily use such a curve to illustrate the concept maximum. We must illustrate by some special instance since any abstract concept is just exactly a set or class of concrete cases. Just as we have shown in the case of a maximum, all mathematical concepts grew out of experience before they were initiated into the fraternity of mathematics by being named maximum, minimum, sine, cosine, differen-

tiation, integration, mathematical induction or whatever appellation the namer chose to give them.

Mathematics, then, is just a collection of concepts that have to do with size and shape, each of which is a class of percepts that possess some common property and are resultants of sensory experiences. It is, therefore, just as natural as any science and can be taught as humanistically. Its distinctive feature is its concise, condensed x, y, z language. But this no more constitutes our science than the language in which a young man declares his love constitutes that love.

It follows from this definition of mathematics that mathematical truth is neither eternal, whatever that may mean, nor definable, whatever that may mean, but is that in which we have experiential faith. It is such a pragmatic test extending over a period of twenty-eight years that has rebuilt my faith, which modernistic tendencies had almost destroyed, in the somewhat old-fashioned thesis of this paper. This rather personal review of that experience has been written in the hope of interesting other mathematics teachers in testing this thesis in their own teaching.

AN INTERESTING LOCUS PROBLEM

By W. E. BYRNE

In an elementary course it may not be desirable to discuss completely many locus problems. However there are some problems which lend themselves quite readily to a complete treatment. Each textbook in analytic geometry should give an explanation of several such problems to familiarize the student with the precautions necessary in investigating new problems.

Let it be a question of finding the locus of points M in a plane (P) such that the angle AMB shall be a given constant w, $0 \ngtr w \ngtr \Pi$. A, B being fixed points in the plane (P). Introduce a system of rectangular coordinate axes such that A, B have the coordinates $(-a,0)$, $(a,0)$, respectively, $(a > 0$ for sake of argument).

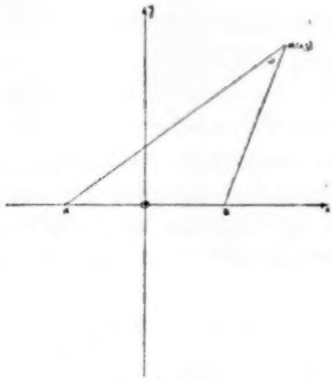


Figure 1

By the law of cosines

(1) $$AB^2 = AM^2 + BM^2 - 2AM.\ \ BM \cos w$$

(2) $$4a^2=(x+a)^2+y^2+(x-a)^2+y^2-2\sqrt{(x+a)^2+y^2}\sqrt{(x-a)^2+y^2}\cos w$$

or

(3) $$x^2+y^2-a^2=\sqrt{(x+a)^2+y^2}\ \sqrt{(x-a)^2+y^2}\cos w$$

If $\cos w=0$, every point M of the locus is on the circle of equation

(4) $$x^2+y^2=a^2$$

If $\cos w\neq 0$, squaring (3) leads, after reduction to

(5) $$(x^2+y^2-a^2)^2 \sin^2 w=4a^2y^2\cos^2 w$$

If $\sin w=0$, (5) gives $y^2=0$ (6)

If $\sin w\neq 0$, (5) may be factored into two equations, each one of which represents a circle passing through A and B:

(7) $$x^2+y^2-a^2=2ay \cot w$$

(8) $$x^2+y^2-a^2= -2ay \cot w$$

Discussion

Case 1. $\cos w=0$. Equations (2) and (4) are equivalent. The points A, B are not points of the locus (in this as well as in the following cases) since in (1) it is assumed tacitly that A, B, M are distinct points. The locus is then the circle of diameter AB, with A, B, deleted.

Case 2. $\sin w = 0$, $\cos w = 1$, $w = 0$.

$y = 0$ combined with (3) shows that

(9) $\qquad x^2 - a^2 = |x^2 - a^2| \qquad$ i. e. $x^2 > a^2$

(with equality excluded since A, B are not points of the locus). The locus is made up of points of the line AB such that either $x > a$ or $x < -a$. The appearance of $y^2 = 0$ in (6) may be explained by noting that when $w \to 0$ the two circles (7), (8) flatten into the x axis.

Case 3. $\sin w = 0$, $\cos w = -1$, $w = \Pi$

(3) gives

(10) $\qquad x^2 - a^2 = |x^2 - a^2|(-1) \quad$ i. e. $\quad a^2 > x^2$

The locus is made up of the interior points of the segment AB.

Case 4. $\sin w \neq 0$, $\cos w > 0$

(3) gives

(11) $$x^2 + y^2 - a^2 > 0$$

so the locus is that portion of the circles (7), (8) outside the circle (4).

Case 5. $\sin w \neq 0$, $\cos w < 0$

(3) gives

(12) $$x^2 + y^2 - a^2 < 0$$

and the locus is that part of the circles (7), (8) inside the circle (4).

In the above discussion only the magnitude (not the sign) of the angle AMB was considered. There is still another question to be answered. What is meant by the angle AMB? If the angle is considered as the *oriented angle* between the *oriented* lines MA, MB, then the restriction $0 \ngtr w \ngtr \Pi$ must be lifted; the knowledge of $\cos w$ will not suffice to distinguish between w and $-w$. If $-\Pi \ngtr w \ngtr \Pi$, each case must be subdivided into two parts according to the sign of w. Again if AMB is taken as the *oriented angle* between the *non-oriented lines* MA, MB the results are entirely different (see Johnson, *Modern Geometry*). The drawings for all cases and the discussions for these special interpretations of angle are left to the reader.

The complete solution of this problem is not at all beyond the powers of the average student of analytic geometry. It illustrates the necessity of checking the equivalence of equations and also the frequently neglected fact that $\sqrt{x^2} = |x|$ when the radical is used to indicate the principal square root. The same problem could be used to advantage as a locus problem in elementary plane geometry. After all is not the classification of all possibilities and the corresponding discussion somewhat neglected in elementary mathematics these days?

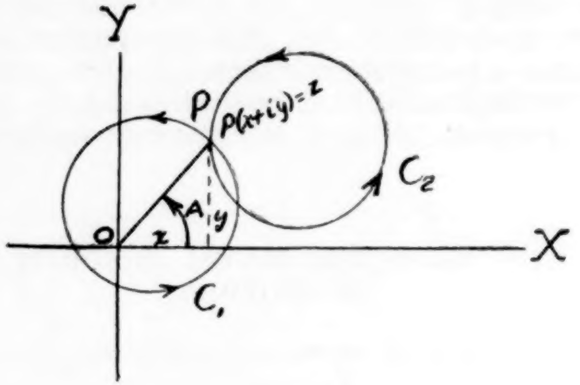
ON THE FUNDAMENTAL THEOREM OF ALGEBRA

By W. PAUL WEBBER

1. Introductory. In many, if not most of the texts on the theory of equations the proof of the fundamental theorem is either treated by some method unfamiliar to students or is relegated to the end of the book. I have been told by students that in such courses they are told that the proof is difficult and rather "beyond the scope" of the course and is therefore omitted, the fundamental theorem being assumed to be true. This may be the correct thing to do nowadays, but I believe if I were teaching the course I could get the class to do a little better. Whether I can write out briefly a course of procedure that is intelligible to readers is another question. No great knowledge of the complex variable is needed if what is required is made clear.

The graphic representation of complex numbers is credited to Argand whose method was published anonymously in *Essai sur une maniere de representier les quantities imaginaires dans constructiones geometriques* in 1806. Some years later this method appeared in *Gergonnes' Annales*. Little attention was given this powerful method for some years, and it is thought that in the meantime similar methods were discovered by Warren in England and by Mourey in France. The method of Argand was published by Gauss in his works in 1831. Gauss used it to prove the fundamental theorem of Algebra. An outline of two proofs of the fundamental theorem based on Argand's representation will be given here. Before doing this a brief exposition of Argand's method of representing complex numbers on a plane will be given.

2. Argand's Graphic Representation of Complex Numbers. The complex number $z=x+iy$ is represented on the complex number



plane by the point whose Cartesian coordinates are x, y. From the figure we see that $OP=\sqrt{x^2+y^2}=r$, say. This is the magnitude of the complex number $x+iy$. The angle A is the angle of the complex number $x+iy$. We see that $\sin A=\dfrac{y}{\sqrt{x^2+y^2}}$, $\cos A=\dfrac{x}{\sqrt{x^2+y^2}}$, and that $x+iy=r(\cos A+i\sin A)$. We shall refer to the point $x+iy$ as the point z.

We see that if z traces a closed curve C_1 about O the angle A will be changed by 2Π, for the line OP will swing through the angle 2Π as in C_1. If z traces a closed curve C_2 without including O the angle A returns to its original value, for OP merely swings back and forth and returns to its starting position.

A rational integral function of z is obtained by applying a finite number of the fundamental operations of Algebra to the variable z. Such a function may be written as

$$f(z)=a_0z^n+a_1z^{n-1}+\ldots+a_{n-1}z+a_n$$

where $a_0,a_1,\ldots$ are complex numbers or real numbers and n a positive integer. When we substitute $x+iy$ for z and carry out the calcu-

lations we shall obtain in general another complex number so that we may write

$$f(z) = w + iv$$

Where w, v are each functions of x,y.

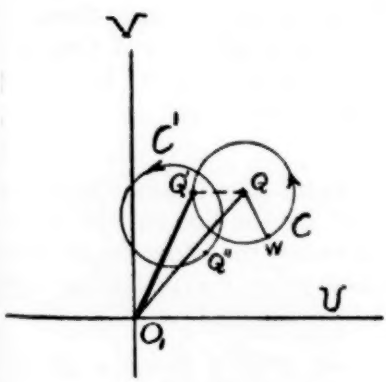
Now f(z) being a complex number admits the same graphic representation that z does. It is customary to make this representation on a separate plane from the z−plane and to call it the w−plane.

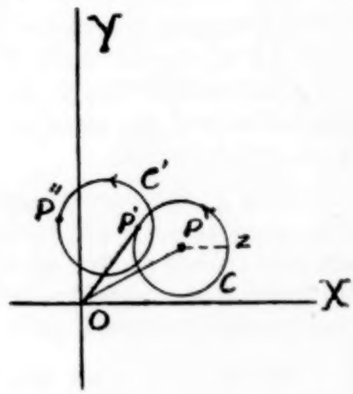
We note here that a rational integral function of z is continuous. Hence,

$$|\Delta W| = |f(z_1 + \Delta z) - f(z_1)| \to 0$$

as $\Delta z \to 0$.

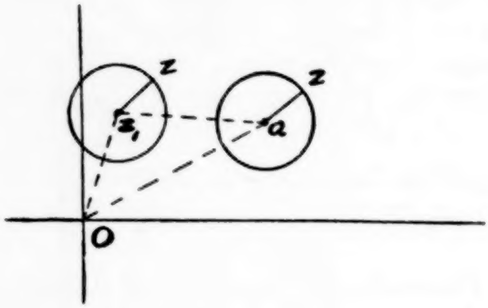
3. The Fundamental Theorem of Algebra. First Proof. Let $w = f(z)$ be a rational integral function of z. Let the point Q in the w-plane correspond to the point P in the z-plane. In general $\Delta w \neq 0$,





if $\Delta z \neq 0$, As z goes around c, w will go around the corresponding curve C. For simplicity these curves may be circles so that c has its center at P and radius Pz while C has its center at Q and radius Qw. As z traverses c there is some position of z on c, say *p'*, such that the corresponding point Q' is nearer to O_1 than is Q. This means that $O_1Q' < OQ$. Note that on account of the continuity of f(z) the radius Pz can be taken so small that the circle C will not enclose the point O_1. With P' as a center another circle c' can be drawn such that the corresponding circle C' about Q' as a center will pass still nearer to

O_1. For some point say P'' on c' a point Q'' will lie nearer to O_1 than does Q'. Now as O_1Q, O_1Q', O_1Q'' are the successive absolute values of $f(z)$ for the corresponding points P, P', P'', by repeating the process we may ultimately make $O_1Q^{(n)}$ less than any assignal quantity. Its limit is zero. Hence, $f(z) = 0$ has a root. **Second Proof.** In this



case it is proposed first to show that the angle of $f(z)$ is changed by 2Π when and only when z makes a closed circuit about some point a which represents a root of $f(z) = 0$. We know by the factor theorem that if a is a root of $f(z) = 0$ then $z - a$ is a factor of $f(z)$ and that if a is not a root of $f(z) = 0$ then $z - a$ is not a factor of $f(z)$. It follows that if $f(z) = 0$ has roots $f(z)$ can be broken into factors of the form $z - a$, and conversely.

Now let $f(z) = z - a$ and let z make a small closed circuit about a. The angle of $z - a$ will be changed by 2Π, for the line za will swing entirely around a. If z makes a circuit about z_1 that does not include a the angle of $z - a$ will return to its original value, for the line zz_1 will merely swing back and forth and return to its original position.

We may say then that when and only when z makes a circuit enclosing a root of $f(z) = 0$ does the angle of $f(z)$ undergo a change of value. Let $f(z) = (z - a_1)(z - a_2)$. Now if z makes a closed circuit enclosing two roots a_1, a_2 of $f(z) = 0$ each line za_1, za_2 will swing entirely around a_1, a_2, respectively, and twice 2Π will be the change in the angle of $f(z)$, and so for any number of roots. We infer that the change of angle produced in $f(z)$ by making z trace a close curve is 2Π times the number of roots included in the circuit.

We know that $f(z) = 0$, where $f(z)$ is a rational integral function, has no infinite roots.

We can therefore make z trace a circuit so large that all roots of $f(z) = 0$, if any, will lie within this circuit. Write

$$f(z) = z^n\left(a_0 + \frac{a_1}{z} + \frac{a_2}{z^2} + \dots + \frac{a_n}{z^n}\right)$$

We now make z trace a circle so large that all roots, if any, of the equation $f(z) = 0$ are enclosed, and at the same time $1/z$ will trace a curve so small that no root will be included within it. Since the above equation is an identity the change of angle of $f(z)$, if any, produced by making z trace a closed circuit will be the same as the change of angle of z^n. This is obviously $2\Pi \cdot n = 2n\Pi$, for $z^n = z \cdot z \cdot \ldots$ to n factors, and as each z goes around its circuit it will have its angle changed by 2Π and z^n will have its angle change by $2n\Pi$. The number of roots of $f(z) = 0$ in the whole plane is therefore $2n\Pi \div 2\Pi = n$.

Note that the first proof established the existence of one root only, while the second proves the existence of n roots. The second proof is called Cauchy's proof.

A CURIOUS FUNCTION FROM ELEMENTARY CALCULUS

(Presented before the Alabama Academy of Science, Birmingham on March 11, 1933.)

By WILLIAM SELL
University, Ala.

The indefinite integral

$$y = f(x) = \int u^x \, du$$

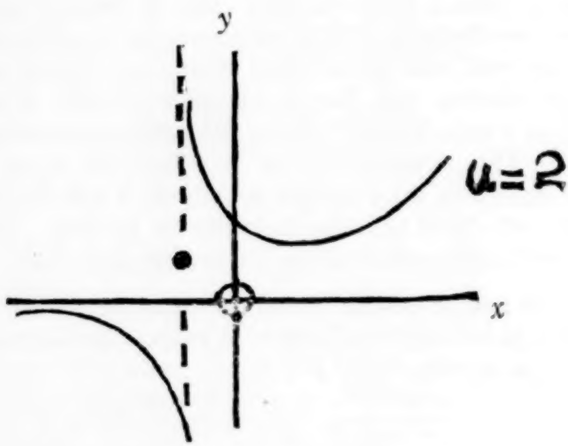
is defined as

$$y = \frac{u^{x+1}}{x+1} \qquad (x \neq -1)$$

$$y = \log u \qquad (x = -1).$$

For a fixed positive value of u, the graph of $f(x)$ somewhat resembles the equilateral hyperbola, having the vertical line $x = -1$ for an asymptote. For $u > 1$, however, the curve rises toward the right. In

addition to two curved branches, there is, by definition, an isolated point $(-1, \log u)$ which completes the locus. It is interesting to know that the ordinate of this point may be thought of as the "average" of two "infinite" ordinates at $x=-1$.



Consider the ordinates at $x=-1+e$ and at $x=-1-e$. The average of these two ordinates is

$$A = \frac{u^e - u^{-e}}{2e} = \frac{1}{e}\,\frac{E^{e\log u} - E^{-e\log u}}{2}$$

$$= \log u \,.\, \frac{\sin h\,(e \log u)}{e \log u}$$

If e approach zero, so too will e log u, and the limit of the second factor is 1. Hence

$$\mathrm{Lim}_{e \to 0}\, A = \log u.$$

A necessary condition for an isolated point of $F(x,y)=0$ is that $F_x = F_y = 0$. $y=f(x)$ may be written in the form

$$F(x,y) = xy + y - u^{x+1} = 0,$$

whence

$$F_x = y - u^{x+1} \log u = 0 \text{ and } F_y = x + 1 = 0.$$

Solving these two equations we obtain $x = -1$ and $y = \log u$ for the coordinates of a possible isolated point.

A PROBABILITY PROBLEM

By S. T. SANDERS, Jr.
St. Joseph, Mo.

The determination of the probability of winning at "craps" affords an opportunity for a neat use of mathematical symbolism. In particular, it would seem that the summation sign could be attractively introduced to a class in this manner:

Pausing to explain the game: the player may win in either of two ways:

(1) by making an initial throw of 7 or 11 (the sum on two dice);

(2) by making an initial throw of 4, 5, 6, 8, 9, or 10 and repeating before a 7 appears.

Let us denote by P, the required total probability;

P_i, the chance of winning with an initial throw of i;

p_i, the chance of throwing an i at any turn;

$q_{i,7}$ the chance of throwing neither an i nor a 7 at any turn;

$P_i^{(j)}$, the chance of winning on the j-th throw with an initial throw of i.

That is, $P_i^{(j)}$ gives the probability of occurrence of the following succession of events:

(1) an initial throw of i;

(2) a throw of neither an i nor a 7 at the second, third, ..., $(j-1)$th turns;

(3) a throw of i at the j-th turn.

There follows immediately

$$P_i^{(j)} = p_i q_{i,7}^{j-2} p_i = p_i^2 q_{i,7}^{j-2}$$

Now evidently $P_i = \sum_{j=2}^{\infty} P_i^{(j)} = \sum_{j=2}^{\infty} p_i^2 q_{i,7}^{j-2} = \sum_{n=0}^{\infty} p_i^2 q_{i,7}^{n}$

And since $P = p_7 + p_{11} + \sum_{i=i'} P_i (1' = 4, 5, 6, 7, 8, 9, 10)$

we have $P = p_7 + p_{11} + \sum_{i=i'} \sum_{n=0}^{\infty} p_i^2 q_{i,7}^{n} (i' = 4, 5, 6, 7, 8, 9, 10)$

Considering the several ways of obtaining the sum, i, from two dice, we form the table:

i	2	3	4	5	6	7	8	9	10	11	12
f_i	1	2	3	4	5	6	5	4	3	2	1
p_i	$1/36$	$1/18$	$1/12$	$1/9$	$5/36$	$1/6$	$5/36$	$1/9$	$1/12$	$1/18$	$1/36$

The symmetry of the table about i = 7 simplifies the formula to

$$P = p_7 + p_{11} + 2 \sum_{i=4}^{6} \sum_{n=0}^{\infty} p_i^2 q_{i,7}^{n}$$

Substituting the values for p_i and $q_{i,7}$ obtained from the table, we have merely to sum the infinite geometric series and obtain

$$P = .493$$

The student might be interested in finding similarly the probability of losing at the game.

BOOK REVIEW DEPARTMENT

Edited by P. K. SMITH

Differential and Integral Calculus. By J. H. Neelley and J. I. Tracey. Published by the Macmillan Company, 1932.

This text is an attempt to serve the needs of both academic and engineering students. The first two chapters, which cover seventy-

five pages, are devoted to Plane Analytic Geometry, and, preparatory to partial differentiation, chapter VIII is devoted to solid Analytic Geometry. The amount of analytic geometry given is sufficient to put a student into calculus; however, it gives him very little reserve knowledge.

The course in Calculus is well-rounded and full. The usual topics found in a first course in Calculus are covered. In addition, in chapter XIX hyperbolic functions are treated in ten pages, and in chapters XX and XXI fifty-three pages are devoted to differential equations and their physical and engineering applications.

The language of the text is clear and forceful. The definition of the derivative and its geometrical and physical interpretations are especially good. Heavy type is used for emphasis. A fair number of exercises and problems are given with each topic, and at the end of many chapters a set of miscellaneous problems are given.

The authors attain a critical spirit in the treatment of the definite integral as the limit of a sum. The question may be raised whether the introduction of the infinitesimal expression of higher order isn't too tedious for engineering students, since the sum may be gotten intuitively without writing out the infinitesimal expression of higher order.

On the whole the figures are splendid. A few exceptions occur, viz., on pages 336, 337, 339, 340, and 347. The shading on some of them might have been left off with better effect. A diagram is the representation usually needed in mathematics—not so much a picture effect.

The text will serve the needs of the engineer better, most likely, than those of the academic student. Two years (three hours per week) would be required to cover the material thoroughly, and, then little time would be available for intensive study on any particular topic.

The mechanical make-up is fine, the paper is thin, and the book compact. The dimensions are 5″ by $7\frac{1}{2}$″. The text contains 496 pages and the list price is $4.00.

PROBLEM DEPARTMENT

Edited by
T. A. BICKERSTAFF
University, Miss.

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and solve problems here proposed.

Problems, and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

Solutions

No. 32. Proposed by T. A. Bickerstaff:

One barrel contains a mixture of $\frac{2}{3}$ wine and $\frac{1}{3}$ water while a second contains a mixture of $\frac{1}{4}$ wine and $\frac{3}{4}$ water. How much must be drawn from each to fill a 2 gallon pail with a mixture of half wine and half water?

Solved by Lucille G. Meyer, New Orleans, La., and Hobson M. Zerbe, Wilkes-Barre, Pa.

Let $x =$ No. of gallons taken from first barrel and $2-x =$ No. of gallons taken from second barrel.

Then $\frac{2}{3}x+\frac{1}{4}(2-x)=\frac{1}{3}x+\frac{3}{4}(2-x)$

From which $x=1^1/_5$ gallons

$2-x= {}^4/_5$ gallons.

No. 29. Proposed by H. T. R. Aude, Colgate University; solved by B. V. Temple, Louisiana College.

Two circles each of radius r move so that their centers stay on a fixed line. Find the area, H, common to the two circles, in terms of s, the distance between their centers, as s varies from 2r to 0.

Solution: Let the two circles whose centers are 0 and 0′ be represented by figure 1 following, where OO′ equals s and varies from 2r to 0. Let A be one intersection of the circles and AB be drawn perpendicular to s. Also let M be the intersection of the circle whose center is 0 on s and θ the angle MOA, measured in radians.

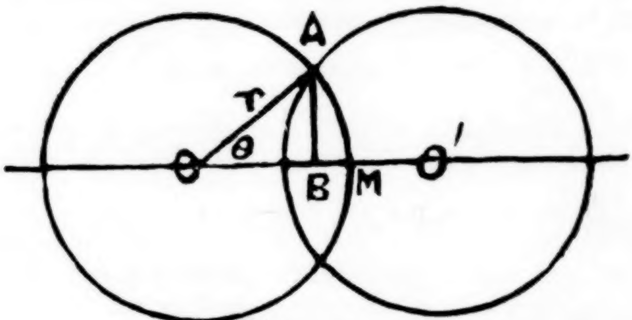


Fig. 1

Now the area, H, common to the two circles is evidently equal to four times the area of ABM.

The area of ABM equals area MOA minus area BOA and therefore

$$H = 4(\tfrac{1}{2}r^2\theta - \tfrac{1}{2}r^2\sin\theta\cos\theta)$$

$$= 2r^2(\theta - \sin\theta\cos\theta).$$

On substitutions from the figure and on simplifying we have

$$H = \tfrac{1}{2}(4r^2 \text{arc cos } s/2r - s\sqrt{4r^2 - s^2}).$$

Also solved by Prof. William V. N. Garretson, Oklahoma A. & M., Stillwater, Okla.

No. 27. Proposed by H. T. R. Aude, Colgate University; solved by V. B. Temple, Louisiana College.

Show that the equation

$$(1)\ 2xy - px - py = 0$$

where p is any prime number greater than 2 has three and only three positive integral solutions.

Solution: Write the equation, (1), in the form

$$(2)\ (2x - p)(2y - p) = p^2.$$

Now since p is a prime number the right member of (2) can be expressed by integral factors only as

$$(a)\ p \,.\, p \text{ or } (b)\ p^2 \,.\, 1.$$

But the left member of (2) is also expressed by only two integral factors, since x and y are to be integral solutions.

Therefore taking the right member as (a) we have

$$(2x - p)(2y - p) = p \,.\, p$$

where we can set

$$2x - p = p \text{ and } 2y - p = p,$$

getting $x = p$ and $y = p$.

Again, on taking the right member as (b) we have

$$(2x - p)(2y - p) = p^2 \,.\, 1$$

where we can set

$$2x - p = p^2 \text{ and } 2y - p = 1,$$

getting $$X = \frac{p(p+1)}{2} \text{ and } y = \frac{p+1}{2}.$$

It is obvious that if we set $2x - p = 1$ and $2y - p = p^2$ we will get no more solutions. Hence there are three and only three solutions namely

$$p\,,\quad \frac{p+1}{2}\,,\quad \frac{p(p+1)}{2},$$

and these are positive, integral and distinct if p is any prime number greater than 2.

Problems for Solution

No. 34. Proposed by R. W. Mattoon, Antioch College, Yellow Springs, Ohio:

Given that two of the angle bisectors of a triangle are equal, to prove by Plane Geometry that the triangle is isosceles.

No. 35. Proposed by Earl Thomas, L. S. U.:

The ends of two ladders rest at the bottoms of parallel vertical walls on opposite sides of an alley and each leans against the wall on the opposite side of the alley. The first is 40 feet long, the second is 30 feet long and they cross at a point 20 feet above the alley. How wide is the alley?

No. 36. Proposed by William Sell, University, Ala.:

Given points A and B on the circumference, and the line l tangent, construct the circle using ruler and compasses.

No. 37. Proposed by William Sell, University, Ala.:

Given point A on the circumference, line l tangent, and line c containing the center. Construct the circle using ruler and compasses.

EDITORIAL PERSONNEL OF THE MATHEMATICS NEWS LETTER

(*Continued from page two*)

W. E. BYRNE

Undergraduate work at Rensselaer Polytechnic Institute, 1915-17, 1919-21 (in service with A. E. F., 1917-19). Degree E. E. June 1921.

Graduate Work

Faculte des Sciences, University of Paris, 1921-22.

Russell Sage Fellow, Rensselaer, 1922-24.

Ph.D., 1924. Thesis on elementary solution of partial differential equations of the parabolic type.

American Field Service Fellow, mathematics, 1924-26.

(Faculte des Sciences, College de France, Ecole Normale Superieure.)

Teaching

Instructor, Princeton University, 1926-28.

Head of Department of Mathematics, Morris Harvey College, Barboursville, West Virginia, 1928-29.

Head of Department of Mathematics, Mississippi Woman's College, Hattiesburg, Mississippi, 1929-31.

Lieutenant-Colonel and Associate Professor of Mathematics, Virginia Military Institute, 1931-.

Membership

Sigma Xi (1921).

Mathematical Association of America (1929).

American Mathematical Society (1924)

Chi Beta Phi Scientific Society (Associate Editor, Mathematics, Chi Beta Phi Record).

Reviews published in the Bulletin and the Monthly and reviews and short articles in the Chi Beta Phi Record.

Present teaching almost entirely differential equations and advanced calculus for engineering students, who have either three or six semester hours of mathematics beyond the calculus.

WILLIAM VAN PARKER

Was born at Monroe, North Carolina, December 22, 1901. Attended University of North Carolina, 1919-24, A. B. 1923, M. A. 1924. Was student assistant (part time instructor) 1922-23, 1923-24. Received Archibald Henderson prize for Master's thesis, 1924. Assistant Professor, University of the South, Sewanee, Tenn., 1924-25. Assistant Professor University of North Carolina, 1925-31. On leave as part time instructor and graduate student at Princeton University, 1926-27. On leave studying on a Fellowship at Brown University, 1929-30, Ph.D., 1931. In addition to papers in the News Letter, had five papers published in the Bulletin of the American Mathematical Society—one in 1931 and four in 1932. Work at Brown was directed by Professor A. A. Bennett. Title of dissertation "Addition Formulae for Hyperelliptic Functions". This was published in two parts in the Bulletin. The first part was "On the Kummer Surface". Present position: Professor of Mathematics, Mississippi Woman's College, 1931-.

THE RUSTON MATHEMATICAL MEETINGS

Reported by
DOROTHY McCOY
Secretary M. A. of A. Section

That there is an interest in mathematics and the teaching of mathematics in Louisiana and Mississippi cannot be questioned after the excellent attendance at the meetings in Ruston, Louisiana, March 3, 4, in spite of banking conditions. There was a good audience from the opening number on the program of the Branch Friday afternoon through the business session of the M. A. of A. Saturday morning.

All details for entertainment were efficiently cared for by the local committee, giving visiting members plenty of opportunity for committee meetings and other valuable though less formal group discussions.

The program itself contained both informational and inspirational materials with the climax at the very last of the business session when the state of Arkansas was invited to join with Louisiana and Mississippi to form tri-state sections. This invitation was to be carried by Dr. Harding who gave two excellent lectures.

Program of the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics

A Play—The Evolution of Numbers—Students of the Training School of Louisiana Polytechnic Institute.

How Many Units Have You?—Mr. James Beckett, Central High School, Jackson Mississippi.

Harmonic Ranges—Mr. Henry Schroeder, Training School of Louisiana Polytechnic Institute.

The Effective Rate Corresponding to a Discount of One Per Cent Per Month as Viewed by the Business Man Himself—Dr. Irby C. Nichols, Louisiana State University.

Officers for the Branch for the coming year are as follows: Chairman, Mr. Henry Schroeder; Secretary, Mr. James Beckett.

Program of the The Louisiana-Mississippi Section of the Mathematical Association of America

Education for the New Era—Dr. A. M. Harding, University of Arkansas.

The Complex Complete Quadrangle—Professor B. E. Mitchell, Millsaps College.

Report on a Special Problem in Research—Dr. Irby C. Nichols, Louisiana State University.

After the banquet on Friday evening Dr. Harding gave an illustrated lecture on "The Depths of Space".

Newly elected officers of the M. A. of A. Section are: P. K. Smith, Louisiana Polytechnic Institute, Chairman; W. V. Parker, Mississippi Woman's College and J. A. Hardin, Centenary College, Vice-Chairmen.

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